Inexact Newton and Interior Point Methods

L. Bergamaschi G. Zilli*

Unità operativa di Padova

In this communication we present the results obtained in the solution of sparse and large systems of non-linear equations by Inexact Newton [3] and Quasi-Newton methods combined with an iterative row-projection linear solver of Cimmino-type [1, 6]. Under weak regularity conditions for F, it is proved that also the Inexact Quasi-Newton method has a local, linear convergence in the energy norm induced by the preconditioned matrix HA, where A is an initial guess of the Jacobian matrix, and it may converge superlinearly too. The matrix $H = [A_1^+, \ldots, A_i^+, \ldots, A_p^+]$, where $A_i^+ = A_i^T(A_iA_i^T)^{-1}$ is the Moore-Penrose pseudo inverse of the $m_i \times n$ block, A_i is the preconditioner. The p linear least squares subproblems (underdetermined) in the pseudoresidual unknowns $\delta_{i,k}$

$$A_i\delta_{i,k} = (b_i - A_i s_k)\,, \quad 1 \le i \le p$$

must be solved at *each* conjugate gradient iteration ($k = 1, 2...$). We first solve them *concurrently* with the iterative Lanczos algorithm LSQR. A simple partitioning of the Jacobian matrix was used for solving a set of nonlinear test problems with sizes ranging from 1024 to 131072 on the CRAY T3E under the MPI environment. To overcome the problem of the costly solution of the least square subproblems, we adopt a suitable block row partitionings of the matrix A in such a way that $A_iA_i^T = I, \quad i = 1, \ldots q$, and consequently, $A_i^+ = A_i^T$. at each step of the inner iteration. This partitioning [5] is always possible for every sparse matrix and produces a number q of blocks A_i whose rows are mutually orthogonal [2].

Our methods may be used to solve more general problems as the non linear mixed complementary problems [4] (including linear and non linear programming problems, variational inequalities etc.). It is indeed known that the interior point solution of a mixed complementary problem can be viewed as a variant of an Inexact Newton method applied to a particular system of non linear equations.

As an example we solve the obstacle Bratu problem [4]. We propose the solution of the linearized system via the Schur complement which reduces the original $5n \times 5n$ system in an equivalent one having only n unknowns. This reduced system is finally solved by an iterative method. We provide numerical results in both sequential and parallel implementations.

References

[1] L. Bergamaschi, I. Moret, G. Zilli, *Inexact quasi-Newton methods for sparse systems of nonlinear equations*, J. Fut. Gen. Comput. Sys. (1999), submitted.

[2] L. Bergamaschi, G. Zilli, *Parallel inexact Newton and interior point methods*, in Parallel Computing 99 (1999), to appear.

[3] R.S. Dembo, S.C. Eisenstat, T. Steihaug, *Inexact Newton methods*, SIAM. J. Numer. Anal. **19**, pp. 400–408 (1982).

[4] S.P. Dirske, M.C. Ferris, *MCLIB: A collection of nonlinear mixed complementary problems*, Tech. Rep. CS Depth., University of Winsconsin, Madison, WS (1994).

[5] G. Zilli, *Parallel implementation of a row-projection method for solving sparse linear systems*, Supercomputer **53**, n. X-1, pp. 33–43 (1993).

[6] G. Zilli, L. Bergamaschi, *Parallel Newton methods for sparse systems of nonlinear equations*, Rendiconti del Circolo Matematico di Palermo Serie II **58**, pp. 247–257 (1999).

*Dipartimento di Metodi e Modelli Matematici per le scienze Applicate, Università di Padova, {berga,zilli}@dmsa.unipd.it.