The numerical treatment of hypersingular integral equations

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It is only in the last years that engineers and applied mathematicians have started to employ finite-part integrals to formulate as boundary integral equations several two and three-dimensional boundary value problems, particularly in the area of applied mechanics. These equations are usually called "hypersingular", since their kernels have a singularity of order greater than the dimension of the integral. We have considered collocation and quadrature methods for solving a singular integro-differential equation of Prandtl's type

$$g(x)v(x) - \frac{1}{\pi}\int_{-1}^{1}\frac{v'(t)}{t-x}dt + \frac{1}{\pi}\int_{-1}^{1}h(x,t)v(t)dt = f(x),\ |x|<1 \tag{1}$$

where the unknown function v(x) has to fulfil the additional conditions

$$v(-1) = v(1) = 0. \tag{2}$$

Several authors have studied this type of integro-differential equations and related numerical methods (see for example [3]). For a function $v \in L^p(-1,1)$ possessing a generalized derivative $v' \in L^p(-1,1)$, equation (1) together (2) can be written in the form

$$g(x)v(x) - \frac{1}{\pi}\int_{-1}^{1}\frac{v(t)}{(t-x)^2}dt + \frac{1}{\pi}\int_{-1}^{1}h(x,t)v(t)dt = f(x),\ |x|<1 \tag{3}$$

where the first integral has to be understood in the sense of finite-part integral. We have proved optimal convergence rates for collocation and quadrature methods for equations(3) in weighted Sobolev norms. Further, we have found a fast algorithm for the numerical solution of(3) based on the quadrature method. Also the case of weakly singular pertubation kernels h(x,t) has been investigated [1]. On the other hand, we have also investigated a regularized version of equation (1) in a scale of weighted Besov spaces as subspaces of weighted spaces of continuous functions in order to obtain convergence results for the collocation and quadrature approximation methods in uniform norms [2].

References

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