Numerical integration of a model for the marine bacteriophage infection influenced by stochastic perturbations

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In a recent paper [1] Beretta and Kuang proposed and analyzed a mathematical model for the marine bacteriophage infection as the system of non linear ordinary differential equations (in dimensionless form),

$$\begin{aligned} \frac{ds}{dt} &= as(1-(i+s)) - sp \\ \frac{di}{dt} &= sp - li \\ \frac{dp}{dt} &= -sp - mp + bli \end{aligned} \tag{1}$$

where s are the susceptibles bacteria, i the infected bacteria and p the phages (viruses), with $s(0) > 0$, $i(0) > 0$, $p(0) > 0$. In that paper it was shown that the behaviour of (1) mainly depends on the 'virus replication factor' $b \in (1, +\infty)$. In order to study the effects of stochastic disturbances on the stability properties of the positive equilibrium of (1), $E_+ = (s^*, i^*, p^*)$, when it is feasible, it is useful to consider the perturbed model with multiplicative noise:

$$\begin{aligned} ds &= (as(1-(i+s)) - sp)dt + \sigma_1(s - s^*)d\xi_1 \\ di &= (sp - li)dt + \sigma_2(i - i^*)d\xi_2 \\ dp &= (-sp - mp + bli)dt + \sigma_3(p - p^*)d\xi_3 \end{aligned} \tag{2}$$

where ξ_1, ξ_2, and ξ_3 are independent from each other standard Wiener processes. In this paper the pathwise (strong) approximation of (2) with respect to the global error in the L_2-norm is considered by means of the Euler-Maruyama and the Milstein methods. Both an equidistant discretization of the time interval and an adaptive step-size control are applied [3]. Results of numerical experiments for all techniques are finally discussed.

References

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