Lie group methods and the approximation of the exponential map

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Discretization methods for ordinary differential equations based on the use of matrix exponentials have been known for decades. This set of ideas has come off age and acquired greater urgency recently, within the context of *geometric integration* and discretization methods on manifolds based on the use of Lie-group actions. In this context it is essential to provide a coordinate map from the Lie algebra to the Lie group in order to implement the methods. Very often the coordinate map is choosen to be the exponential map. In the numerical implementation of the methods the exact esponential is substituted by a suitable numerical approximation, carried out to machine precision. Aiming to cheaper implementations of the methods, it is possible to substitute the exponential coordinate map with some alternative coordinate maps. If the Lie group integrator has order-p, one way of obtaining cheaper coordinate maps is simply to replace exp in the ode solver, with suitable order-p numerical approximations. It is then essential that given a Lie group $\mathbf{G}$ and its Lie algebra $\mathbf{g}$, the approximants $F(tB)$ of $\exp(tB)$ are such that $F(tB) \in \mathbf{G}$ if $B \in \mathbf{g}$. We will discuss several possibilities for $F(tB)$, illustrating implementation details and complexity advantages.

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