On the approximate computation of singular and hypersingular integrals

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We have been interested in the numerical evaluation of integrals of functions with strong singularities. Such integrals exist only if defined in a proper sense. The concept of Cauchy principal value integral is well known and often used in applications. Several boundary problems are formulated as singular integral equations involving such integrals. We consider the numerical analysis of the methods for their direct computations in the case of the "finite" Hilbert transform proposing new quadrature rules, proving their convergence and stability. Further, we have improved and specified some previous results on other numerical procedures [4, 6, 7]. On the other hand we have analysed the numerical approximation of the Hilbert transform. Even if the importance of the Hilbert transform coming from its many applications justifies some interest in its numerical evaluation, the literature concerning this problem is very poor. We have proposed an algorithm for the approximate evaluation of the Hilbert transform proving the convergence and the stability of the procedure [1, 2, 3]. Finally, we have related our interest to the less known integrals with stronger singularities, usually called finite-part integrals, studying the numerical approximation of two different kinds of such integrals. In any case the procedures proposed are convergent for a wide class of functions and they are numerically stable. The previous results in the related literature have been improved [4, 5].

References

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