Conservative methods for orthogonal differential systems on the Stiefel manifold

N. Del Buono*

October 13, 1999

In the last years several numerical methods have been proposed for solving matrix differential systems whose solutions evolve on special manifolds, for instance the manifold of the square orthogonal matrices, the symplectic manifold or the manifold of the square oblique rotation matrices.
Here we propose numerical methods for solving ODEs on the Stiefel manifold, that is the set of the $n \times p$ orthogonal matrices, with $p < n$. This manifold represents the constraint that arises in such numerical linear algebra areas as the symmetric eigenvalue problem, nonlinear eigenvalue problems, electronic structures computations, and signal processing.
Our methods, whose computational cost is less than n^3, are based on the Householder decomposition and on the use of the embedded geodesics on the Stiefel manifold.

References

[1] M.P. Calvo, A. Iserles, A. Zanna, *Runge-Kutta methods for orthogonal and isospectral flows*, Appl. Num. Math. **22**, pp. 153–164 (1996).

[2] N. Del Buono, L. Lopez, *Numerical methods for Hermitian unitary differential systems*, J. Comp. Appl. Math., to appear.

[3] N. Del Buono, L. Lopez, *Conservative methods for solving differential sistems on the manifold of the square oblique rotation matrices*, submitted.

[4] F. Diele, L. Lopez, R. Peluso, *The Cayley transform in the numerical solution of unitary differential systems*, Advances in Comp. Math. **8**, pp. 317–334 (1998).

[5] L. Dieci, D. Russell, E. Van Vleck, *Unitary integration and applications to continuous orthonormalization techniques*, SIAM J. Num. Anal. **31**, pp. 261–281 (1994).

[6] E. Edelman, T.A. Arias, S.T. Smith, *The geometric of algorithms with orthogonality constraints*, SIAM J. Matrix Anal. Appl. **20**, pp. 303–353 (1998).

[7] L. Lopez, C. Mastroserio, T. Politi, *Variable step-size tecgniques in continuous Runge-Kutta methods for isospectral dynamical systems*, J. Comput. Appl. Math. **82**, pp. 261–278 (1997).

*Dipartimento Interuniversitario di Matematica, Università degli Studi di Bari, delbuono@gauss.dm.uniba.it.