# Numerical Methods for Structural Mechanics Problems and Non Linear Materials

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We consider and analyse mathematical models, numerical methods and algorithms for problems arising in the mechanics of structures and in the simulation of the mechanical properties of linear and non linear materials. In the following we summarize the topics involved in this work.

First we analyze some finite elements for the numerical solution of Naghdi cylindrical shell problems. Its discretization with standard finite elements suffers of the locking phenomenon, i.e., when small thickness shells are solved, the numerical results are affected by a large error. We have proposed several ways to avoid the numerical locking: mixed formulations, reduced and/or selective integration. The performance of the new elements is tested by solving some benchmark problems involving very thin shells, representative of *membrane* and *bending* dominated situations. To assess the robustness of the finite elements, computations have been carried out with uniform and distorted decompositions of the domain. The numerical results show both properties of convergence and robustness [1, 2, 3, 4, 5, 6, 7, 8, 9, 10].

Often the behaviour of the engineering materials is non linear, since the acting forces bring the material beyond the elastic limit. The classical theory of plasticity is mostly used since it allows a modelling that can be experimentally corroborated. In this framework we have developed a new numerical scheme using wavelet bases and a suitable variational formulation of the physical problem [11, 12]. We show the basic properties of the new numerical approach showing some numerical test both in static and dynamical case. In particular we consider the possibility to state adaptive algorithms for the computation of the plastic wave.

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