Interpolatory quadratures for the numerical evaluation of the weighted Hilbert transform on the real line

M.R. Capobianco* G. Criscuolo† **R. Giova**‡

The Hilbert transform has a variety of applications; the ones to be mentioned here concern casuality and generalization of the phasor idea beyond pure alternating current. Besides appearing in several physical and engineering problems, the Hilbert transform is the main part of singular integral equations on R. The importance of the Hilbert transform coming from its many applications, justifies some interest in its numerical evaluation. The results about the numerical approximation of the "finite" Hilbert transform are several and quite satisfactory, while only a small number of publications, see [1, 2, 3, 4, 5, 6] deal with the numerical evaluation of the Hilbert transform. We will confine our discussion to the numerical integration method for weighted Hilbert transform with respect to the Hermite weight function. A possible method starts from the identity:

$$\int_{-\infty}^{+\infty} \frac{f(x)}{x-t} \exp(-x^2) dx = f(t) \int_{-\infty}^{+\infty} \frac{\exp(-x^2)}{x-t} dx + \int_{-\infty}^{+\infty} \frac{f(x)-f(t)}{x-t} \exp(-x^2) dx, \quad t \in R;$$

hence, assuming that the first integral on the right-hand side can be computed we approximate the second integral by a classical interpolation quadrature rule Q_n. Obviously, from a theoretical point of view, this method turns out to be convergent if the function f is sufficiently smooth. However, from a numerical point of view it doesn't give good results when the knots of the quadrature are very close to the singularity t. We examine the convergence and the stability in the uniform norm of the quadrature rules obtained by making use of the ordinary Gaussian rule or another interpolatory rule based on a special set of knots as rule Q_n.

References

[1] M.R. Capobianco, G. Criscuolo, R. Giova, *Approximation of the weighted Hilbert transform on the real line by an interpolatory process*, submitted.

[2] M.R. Capobianco, G. Criscuolo, R. Giova, *A new algorithm to evaluate Hilbert transform on the real line*, submitted.

[3] M.R. Capobianco, G. Criscuolo, R. Giova, *Product integration to evaluate the Hilbert transform. Rules based on the zeros of the Hermite polynomials*, Prepint Univ. Napoli "Federico II", Liguori Editore (1999).

[4] S.B. Damelin, K. Diethelm, *Interpolatory product quadratures for Cauchy principal value integrals with Freud weights*, Numer. Math. **83**, pp. 87–105 (1999).

[5] M.C. De Bonis, B. Della Vecchia, G. Mastroianni, *Approximation of the weighted Hilbert transform on the real line*, Rapporto tecnico dell'Universitá della Basilicata **1/98** (1998).

[6] K. Diethelm, *A method for the practical evaluation of the Hilbert transform on the real line*, J. Comp. and Appl. Math., to appear.

*Istituto per Applicazioni della Matematica, C.N.R., capobian@iamna.iam.na.cnr.it.

†Dipartimento di Matematica e Applicazioni, Università degli Studi di Napoli "Federico II", criscuo@matna2.dma.unina.it.

‡Dipartimento di Matematica e Applicazioni, Università degli Studi di Napoli "Federico II", giova@matna3.dma.unina.it.