Asymptotic expansions for some classical positive operators and their use in approximation theory

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In this paper we review some results recently introduced by the authors about the use of extrapolation techniques in approximation theory.

We consider some classical positive operators to approximate a given function f and we prov the existence of asymptotic expansion formulas for them. These formulas used in connection with extrapolation techniques are shown to be effective in approximating the given function f, at the same time, fastly and efficiently.

In [2] we consider the classical Bernstein polynomials $B_n(f)$ and we prove the expansion formula with respect to the grid "parameter" $h = n^{-1}$. Therefore in [6, 4] we analyze some interesting applications of these results to the indefinite integral and to the solution of the Cauchy problem.

Similar results are shown in [5] for the De La Vallée Poussin operator.

In [3, 1] we generalize the theory to the set of general exponential type operators and to the modified Bernstein polynomials.

Finally in [7] a global theorem of existence joins all the results.

For every case several numerical experiments fully confirme the theoretical analysis.

References

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