Stability properties of Runge-Kutta methods for delay differential equations

N. Guglielmi*

Unità operativa di Trieste

In this talk we investigate the asymptotic stability properties of natural symmetric Runge-Kutta (R–K) processes for delay differential equations of the form:

$$y'(t) \quad = \quad f\Big(t, y(t), y(t-\tau)\Big).$$

The last decade has seen a relatively large number of papers concerned with the study of the stability of numerical methods, using several test model problems of the previous form. A common feature of those papers is that conditions that are stronger than necessary for the (asymptotic) stability of the solutions are considered. In particular, they typically make assumptions such that the solutions are asymptotically stable for every delay $\tau > 0$.

Very seldom, instead, was taken into consideration the dependence of the asymptotic stability on the delay. Recently, in [1, 2, 3, 4, 5], was developed a general analysis of R–K processes for delay differential equations (DDEs) on the linear scalar constant coefficient model problem $y'(t) = a\,y(t) + b\,y(t-\tau)$, which focused attention on the delay dependence of the asymptotic behaviour of the numerical solutions, with respect to the true solutions.

Such analysis allows for a better understanding of the requirements for DDE methods to be stable but seems to be quite difficult, even on apparently simple model problems. At the moment, in fact, relevant to the abovementioned constant coefficient equation, only for a few classes of methods has been possible to give a complete stability analysis. In particular, the analysis developed here includes the class of one-step Gaussian collocation methods and Radau methods. As far as we know, these are the first classes of high order methods which are proved to preserve the asymptotic stability of the true solutions for the considered model problem.

References

[1] N. Guglielmi, *On the asymptotic stability properties of Runge–Kutta methods for delay differential equations*, Numer. Math. **77**, n. 4, pp. 467–485 (1997).

[2] N. Guglielmi, *Delay dependent stability regions of* Θ*-methods for delay differential equations*, IMA J. Numer. Anal. **18**, pp. 399–418 (1998).

[3] N. Guglielmi and E. Hairer, *Order stars and stability for delay differential equations*, Numer. Math. **83**, n. 3, pp. 371–383 (1999).

[4] S. Maset, *Stability of Runge-Kutta methods for linear delay differential equations*, Numer. Math. (1999), in press.

[5] N. Guglielmi and E. Hairer, *Geometric proofs of numerical stability for delay equations*, (1999), submitted.

*Dipartimento di Matematica Pura e Applicata, Università di L'Aquila, guglielm@univaq.it