Uni and bivariate polynomial spline approximation

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In this paper we consider numerical methods for function representation based on uni and bivariate spline operators.

Although approximation theory of univariate splines has already grown firm and there is a wide literature and high quality software about it, the study of characterising properties and the definition of new schemes are still research object. In this context higher order lacunary spline interpolation is considered and the corresponding error analysis with optimal convergence rates is presented [9, 10]. Moreover both optimal nodal interpolating and quasi-interpolating (q-i) polynomial spline operators are studied with particular reference to their convergence properties and their approximation power [5, 7]. New results about piecewise weighted mean functions for approximating histograms are also proposed in [6].

In many scientific and engineering areas approximating functions in two or more variables are often needed and among them the set of multivariate piecewise polynomial functions, usually called splines, is very useful. The problem of the construction of bivariate splines is in principle equivalent to an algebraic system arising from the "global conformality condition" (gcc). For cross-cut partitions such a system can be divided into independent sub-systems and each one again is partitioned into independent sub-systems [8], solvable by parallel algorithms. In the same context local bivariate quadratic C^1 q-i spline operators, defined on a four directional mesh by using a $B-$spline basis, are studied. Moreover it is shown how well they can approximate a real function and its partial derivatives up to optimal order and local and global upper bounds both for the errors and for the spline partial derivatives are given. Then such general results are applied to prove new properties of two known interesting q-i spline operators [1, 2, 3]. Finally Matlab software for their construction is proposed [4].

References

[1] C. Dagnino, P. Lamberti, *On certain bivariate local spline quasi-interpolating operators*, 18th Biennial Conference on Numerical Analysis, Dundee, June 29-July 2 1999.

[2] C. Dagnino, P. Lamberti, *On the approximation power of bivariate quadratic C^1 splines*, Tech. Rep. **38** (1999), submitted.

[3] C. Dagnino, P. Lamberti, *Risultati teorici e sperimentali sull'approssimazione 2-D di funzioni mediante certi operatori spline polinomiali quasi-interpolanti locali*, XVI Congresso UMI, Napoli, 13-18 Settembre 1999.

[4] C. Dagnino and P. Lamberti, *SPLISURF: un software per la rappresentazione di spline bivariate su triangolazioni di tipo 2*, Internal Report (1999).

[5] V. Demichelis, *Convergence of derivatives of optimal nodal splines*, J. Appr. Th. **88**, n. 3, pp. 370–383 (1997).

[6] V. Demichelis, *Piecewise weighted mean function and histograms*, Rocky Mount. J. Math. **28**, n. 4, pp. 1259–1265 (1998).

[7] V. Demichelis, *Spline approximation with interpolation constraints for numerical integration*, Int. J. Computer Math. **70**, n. 3, pp. 739–750 (1999).

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[8] P. Lamberti, *A parallel approach to bivariate splines*, J. Math. Res. Exp. **17**, n. 2, pp. 361–370 (1997).

[9] G. Pittaluga, L. Sacripante, E. Venturino, *Higher order lacunary interpolation*, Rev. Acad. Canar. Cienc. **IX**, n. 1, pp. 33–46 (1997).

[10] G. Pittaluga, L. Sacripante, E. Venturino, *Lacunary interpolation with a¡rbitrary data of higher order*, Ann. Univ. Sci. Budapest, Sect. Comput., to appear.