Numerical Solution of Delay Differential Equations as Abstract Cauchy problems

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Recently a new approach for the numerical solution of Delay Differential Equations (DDEs), different from the classical step-by-step integration, has been introduced (see [1, 2]). A DDE

$$y'(t) = f(y(t), y(t-\tau)),\ t \geq 0, \qquad y(t) = \varphi(t),\ -\tau \leq t \leq 0 \tag{1}$$

is restated as the hyperbolic Partial Differential Equation (PDE)

$$\frac{\partial U}{\partial t}(t,\theta) = \frac{\partial U}{\partial \theta}(t,\theta),\ t \geq 0,\ 0 \leq \theta \geq -\tau \tag{2}$$

provided with the boundary condition

$$\frac{\partial U}{\partial \theta}(t,0) = f(U(t,0), U(t,-\tau)),\ t \geq 0 \tag{3}$$

and the initial condition

$$U(0,\theta) = \varphi(\theta),\ \theta \in [-\tau, 0], \tag{4}$$

and then, by using a class of schemes, the PDE is discretized in a system of Ordinary Differential Equation (ODE) by the method of lines.

This approach can be useful for solving DDEs in a large integration window with respect to the delay. Indeed, whereas in step-by-step integration we have to proceed with stepsizes no larger than the delay, in the integration of the ODE obtained by the method of lines we can proceed with arbitrary large stepsizes. Moreover, when we consider the preservation of the asymptotic behaviour in the numerical solution, this approach works better than the classic one.

References

[1] A. Bellen, S. Maset, *Numerical solution of constant coefficient linear Delay Differential Equation as abstract Cauchy Problems*, Numerische Mathematik **106** (1999).

[2] S. Maset, *Asymptotic stability in the numerical solution of linear pure delay differential equation as abstract Cauchy problems*, Journal of Computational and Applied Mathematics, in press.

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