Numerical approach to some problems of Mathematical Physics

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We considered an iterative method of calculation relative to the numerical approximation of the eigenvalues of a Fredholm operator of the second type, testing it in some particularly significant cases, including the calculation of the eigenvalues of the so-called Ostrowski kernel (see [1]–[4]).

We studied the possibility of obtaining a general formula for representing the information entropy associated to a system of orthogonal polynomials (see [5]). Recall that this topic is currently being studied intensively by many theoretical physicists (see for example the excellent work by Dehesa *et al.* [13], in which asymptotic formulas for the aforementioned entropy are given and connections with the fundamental problems of quantum mechanics are discussed).

We obtained results relating to the asymptotic distribution of the zeros of polynomials, called relativistic, which are orthogonal with respect to variable weights (see [2]).

We studied the distribution of the zeros of polynomial solutions of higher order linear differential equations with polynomial coefficients (also in the non-hypergeometric case), through the construction of the relative Newton sum rules of the zeros.

The method was then applied to the construction of Newton sum rules for associated and co-recursive of classical orthogonal polynomials (see [11]–[12]). This last result was obtained by utilizing differential equations satisfied by such polynomials which recently appeared in some works of A. Ronveaux – F. Marcellán, A. Ronveaux – S. Belmehdi and A. Zarzo – E. Godoy. The method of calculation makes use of the Lucas polynomials of the first kind, avoiding the complicated steps in the so-called sum rules of K.M. Case.

We studied properties of the Fibonacci polynomials, with particular attention to their interpretation in the field of symbolic dynamics. In particular, we computed the zeta function and the entropy of certain classes of matrices, including some generalizations of the Fibonacci matrix (see [6]–[7]).

We looked at the possibility of obtaining sequences of polynomials or piecewise polynomials uniformly approximating the solution of a Cauchy problem for equations or systems of ordinary differential equations. To this end we employed Chaplygin's method and polynomial approximations using Bernstein polynomials (see [10]).

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