A polynomial method based on truncating Faber series for the computation of functions of unsymmetric matrices

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Given a real nonsymmetric matrix A of order N and a N-dimensional vector v, we consider the problem of the computation of $f(A)v$, where f is a given function that we suppose to be analytic in a certain domain of the complex plane containing the spectrum of A, $\sigma(A)$.
The proposed method is of polynomial type based on the approximation of f through Faber polynomials defined on a certain compact subset of the complex plane containing $\sigma(A)$. In particular, given a certain compact $\Omega \subset \mathbf{C}$ such that $\sigma(A) \subseteq \Omega$, we consider a polynomial approximation scheme

$$p_{m-1}(A)\, v \approx f(A)v,$$

in which the polynomials p_{m-1}, $m \geq 1$, of degree $m-1$, are the truncating Faber series with respect to Ω and the function f.

The main features of the method here proposed are the following:

a) if $\sigma(A) \subseteq \Omega$ and f is analytic in Ω, the method converges. Moreover, depending on the position of the singularity of f outside Ω, the method can converge even if $\sigma(A)$ is not contained in Ω.

b) The use of Faber polynomials ensures the existence of a recurrence relation with a finite number of terms for the approximants $y_m = p_{m-1}(A)\, v$.

c) The definition of the iteration parameters for the above mentioned recursion is independent of v.

d) No explicit computation of any matrix functions must be performed (this is not the case of Krylov type methods).

e) The cost at each step is constant and substantially equal to that of an application of A.

References

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