Nearly defective complex eigenvalues in Bessel matrices

R. Pavani*

Unità operativa di Milano Politecnico

We consider the case of ill-conditioned eigenvalues of the real matrix T_n defined as follows:

$$T_n = \begin{bmatrix} -1 & \sigma_1 & & & \\ -\sigma_1 & 0 & \sigma_2 & & \\ & -\sigma_2 & \cdot & \cdot & \\ & & \cdot & \cdot & \sigma_{n-1} \\ & & & -\sigma_{n-1} & 0 \end{bmatrix}$$

where $\sigma_h = -1/(4h^2-1)^{1/2}, \qquad h = 1, 2, ..., n-1, \qquad n \in \mathbb{N}^+$. It is known that such a matrix exhibits simple complex eigenvalues which are getting more and more ill-conditioned for increasing n. Tha aim of this paper is to study the behavior of such eigenvalues, which are simple in theory, but in practice become defective using finite precision. To this end we have tackled the problem of computing the distance from the Bessel matrix T_n to the nearest defective matrix, using two different approaches. They differ, as the first one provides an estimation of the distance to the nearest defective matrix with an imposed multiple eigenvalue; whereas the second one looks for the closest defective matrix with any multiple eigenvalue; moreover the second approach is of statistical type. We have shown that the second approach provides a little sharper estimation of the distance to the nearest defective matrix, as it is expected. We conclude that for n large enough the matrix T_n becomes so close to a defective matrix that it can be considered numerically defective itself (within the machine precision).

*Dipartimento di Matematica, Politecnico di Milano, rafpav@mate.polimi.it.