Mixed Finite Elements and Finite Volumes for Density Dependent Flow and Transport in Groundwater

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The mathematical model of density-dependent flow in groundwater can be formulated as a coupled system of two partial differential equations, one describing mass conservation for the water-salt solution (the flow equation), and the other mass conservation for the salt contaminant (the transport equation). From a physical point of view, the salt contained in the groundwater affects the solution density and induces changes in the flow field, which becomes dependent not only on the hydrogeological parameters of the aquifer, but also on the salt concentration. This interdependence produces a coupling between the flow and transport equations.

The solution of the discretized system of equations ia usually addressed with an iterative Picard-like scheme by which the problem is decoupled by first solving the flow equation, then calculating the velocity field, and finally solving the transport equation. This three-step sequence is repeated until convergence. A difficulty for the successful application of the above procedure is the requirement that accurate velocity fields be obtained from the solution of the flow equation. Another problem that may influence the convergence of the Picard method is oscillatory behavior of the solution when the transport equation is advection dominated.

To overcome the previous problems, an algorithm is developed, based on the Mixed Hybrid Finite Element (MHFE) method for the flow equation [1], and a combination of MHFE with high resolution Finite Volume (FV) scheme for the transport equation [2]. In the latter approach, discretization in two spatial dimensions is obtained by means of triangle-based lowest order MHFE and TVD (Total Variation Diminishing) FV for the dispersive and advective fluxes, respectively. MHFE and FV are knoknw to be stable and accurate (both are globally second order accurate in space). Combination of these two schemes is implemented via a time-splitting algorithm allowing an explicit time-stepping for FV and an implicit time-stepping for MHFE. The overall procedure is shown to attain first order accuracy in time, even though a second order Runge-Kutta time stepping is used. To resort to global second order accuracy a correction term is added in the FV equation [3]. One dimensional numerical tests on the advection-dispersion equation ascertain the effectiveness of this correction.

The coupled iterative scheme is applied for the solution of Elder's problem, a typical test used in benchmarking density-dependent flow and transport numerical codes. The results indicate that the proposed approach is accurate and reliable, and does not suffer from numerical oscillation and does not introduce a large amount of numerical diffusion, as is typically the case with more conventional upwind schemes. Also, the presence of unsymmetric meshes does not dramatically affect the solution, as reported in the literature for other types of discretization approaches. Finally, an application to the solution of the problem of density driven transport of radionuclides at the Lake Karachai site (South Urals, Russia) is provided.

References

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