Parallel methods for Volterra integral equations

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Evolutory problems with hereditary characteristics, such as population dynamics, propagation of epidemics, and others, are described by large and often highly stiff systems of Volterra Integral Equations (VIE) [8, 9]; moreover most of these problems are high velocity processes.

Therefore an efficient numerical treatment of these problems requires the development of parallel methods with strong stability properties. A significant stability analysis, overcoming the bounds due to the use of the test equation, can be performed only after the development of a general theory of the discrete Volterra equations.

In this research project iterative massively parallel waveform relaxation (WR) methods, which have been recently introduced by some components of the research group for solving Volterra integral equations (VIE), have been developed for solving both VIE with regular kernels and VIE with weakly singular kernels, the latter arising from reaction–diffusion problems in small cells [9].

As far as the VIE with regular kernels are concerned, discrete time WR methods based on linear multistep methods have been developed and analysed. Special care has been devoted to the convergence problem, both when the discretization step goes to zero, having fixed the iteration number, and when the iteration number goes to infinity. Necessary and sufficient conditions and easily applicable sufficient conditions for the convergence of the fully parallel WR methods have been derived [3].

Because of the bounds on the convergence rate of such methods, fast fully parallel WR methods for particular kernels have been developed by using a Chebyshev type acceleration technique [2].

As far as the equations with weakly singular kernels are concerned, both continuous time and discrete time based on collocation methods WR methods have been developed. The convergence analysis with respect to the iterate number has requested a new approach that, taking into account the non-smoothness of the true solution, has allowed to estimate how the singularity slow down the convergence rate [1].

Finally, in order to perform a significant analysis of the numerical stability of a generic method by studying the equation which is satisfied by its global error, conditions on the asymptotic behaviour of the solution of discrete Volterra equation have been derived. These conditions have allowed to perform the stability analysis of some linear methods applied to particular classes of Volterra equations [4, 5, 6, 7].

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