Fast spectral methods for the Fokker-Planck-Landau collision operator of plasma physics

G. Russo[*]

October 13, 1999

In this talk paper we present a new spectral method, developed in collaboration with L. Pareschi and G. Toscani, for the fast evaluation of the Fokker-Planck-Landau collision operator. Here we will mainly concentrate on the approximation of the collision operator, and hence of the velocity space.

The Fokker-Planck-Landau equation (FPLE) is a common kinetic model in plasma physics. In the space homogeneous case, it is described by a nonlinear partial integrodifferential equation

$$\frac{\partial f}{\partial t} = Q_L(f,f), \qquad v \in \mathbb{R}^3, \tag{1}$$

where Q_L is the Landau collision operator and the unknown function $f = f(v,t)$ represents the density of a gas in the velocity space.

The spectral method is derived by approximating the density function by a truncated Fourier series

$$f_N(v) = \sum_{k_1=-N}^{N} \sum_{k_2=-N}^{N} \sum_{k_3=-N}^{N} \hat{f}_k e^{ik\cdot v}, \tag{2}$$

The scheme that we obtain has the form

$$\frac{\partial \hat{f}_k}{\partial t} = \sum_{\substack{l+m=k \\ l_i, m_i=-N}}^{N} \hat{f}_l \hat{f}_m \hat{\beta}_L(l,m), \quad k = (k_i, i=1,\dots,3), k_i = -N,\dots,N, \tag{3}$$

where the right hand side of Eq. (3) can be written as a sum of seven convolutions in three dimensions. This allows a computation of the collision integral in $O(n \log n)$, where $n = (2N)^3$ is the total number of Fourier modes.

Consistency analysis shows that, under the assumption that the functiton $f(v,t)$ decays fast enough in velocity, the scheme is spectrally accurate. Furtheremore, we show that the method preserves the total mass whereas momentum and energy are approximated with spectral accuracy. Numerical results showing the acuracy and efficiency of the method are presented.

[*]Department of Mathematics, University of L'Aquila, `russo@univaq.it`