Parallel Preconditioning of Gradient Eigensolvers

L. Bergamaschi G. Pini* **F. Sartoretto**†

Unità operativa di Padova
Unità operativa di Venezia

We deal with the computation of a small number of the leftmost eigenpairs of $A\boldsymbol{x} = \lambda B\boldsymbol{x}$, where A and B are large, sparse, symmetric positive definite matrices. We analyze the performance of a number of preconditioning techniques when applied to an optimization method, called DACG (Deflation-Accelerated Conjugate Gradient) [1, 2]. DACG sequentially computes a number of eigenpairs by CG minimizations of the Rayleigh quotient over subspaces of decreasing size. When effectively preconditioned [3], the efficiency of DACG well compares with that of established packages. We found that the IC(0)-DACG algorithm is more efficient than Lanczos method, as implemented in ARPACK package, when a small number of eigenpairs (say 10) is to be computed.

In this communication we study FSAI and AINV, falling into the class of *approximate inverse preconditioners.* These preconditioners have been devised by many authors in order to accelerate, in a parallel environment, iterative methods for solving linear systems $A\boldsymbol{x} = \boldsymbol{b}$. These preconditioners explicitly compute an approximation, say M, to A^{-1}, based on the sparse factorization of A^{-1}. Preconditioning by a product of triangular factors performs better than other techniques, mainly because the fill-in of the preconditioner is reduced. Unlike many other approximate inverse techniques, AINV and FSAI preserve the positive definiteness of the problem, which is essential in our application. The FSAI algorithm requires an a priori sparsity pattern of the approximate factor, which is not easy to provide in unstructured sparse problems. The drop tolerance parameter ε of AINV has a strong impact on memory requirements, driving the fill-in of the preconditioner. However, the CPU time spent by AINV-DACG is usually not much influenced by the value of ε. Numerical results point out that the factorized approximate inverse preconditioners represent a valid alternative to the efficient IC(0) for the partial eigenvalue problem, in a sequential environment.

We tried the parallel AINV-DACG and FSAI-DACG algorithms in the solution of Finite Element (FE) and Finite Difference (FD) eigenproblems, both in two and three dimensions. A parallel implementation of the DACG algorithm has been performed via a data-parallel approach and an efficient routine for performing matrix-vector products was designed and implemented. The parallel DACG algorithm turns out to be an efficient code which recorded satisfactory speedup on a Cray T3E machine. The Block-Jacobi preconditioner displays comparable performance with AINV (and FSAI) on FD problems, On FE problems, and particularly on ill-conditioned ones, Block-Jacobi is not competitive with the factorized approximate inverse preconditioners. The speedups scored by AINV-DACG and FSAI-DACG were similar, though for $p = 32$ processors FSAI usually showed a slightly better parallelization level [4]. A final comparison with PARPACK, a parallelized version of ARPACK, shows the (relative) superiority of our parallelized DACG code.

References

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*Dipartimento di Metodi e Modelli Matematici per le Scienze Applicate, Università di Padova, {berga,pini}@dmsa.unipd.it.
†Dipartimento di Informatica, Università di Venezia.