Numerical global optimization methods and local information

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This paper deals with the following well known [1, 2, 6, 7] global optimization problem of finding the global minimum of a function $\Phi(z)$ of N variables over a hyperinterval D:

$$\Phi(z^*) = \min\{\Phi(z) : z \in D\}, \tag{1}$$

$$D = \{z \in R^N : a_j \leq z_j \leq b_j, 1 \leq j \leq N\}, \tag{2}$$

where $\Phi(z)$ is multiextremal. If $\Phi(z)$ is continuous then, for the function

$$\phi(x) = \Phi(z(x)), x \in [a, b], \tag{3}$$

where $z(x)$ is the continuous Peano-type mapping of closed interval $[a, b]$ onto the hyperinterval D, we have [6]

$$\min\{\phi(x) : x \in [a, b]\} = \min\{\Phi(z) : z \in D\}.$$

Therefore, solving the multidimensional problem (1), (2) is reduced to solving the one-dimensional problem

$$\phi(x^*) = \min\{\phi(x) : x \in [a, b]\}. \tag{4}$$

In addition, if $\Phi(z)$ is Lipschitzean with a constant $K > 0$ then $\phi(x)$ satisfies the Hölder condition

$$\mid \phi(x') - \phi(x'') \mid \leq H \mid x' - x'' \mid^{1/N}, x', x'' \in [a, b], \tag{5}$$

with a constant $H > 0$ (Hölder constant). For $N = 1$ we also consider the problem (4) where the first derivative $\phi'(x)$ satisfies the Lipschitz condition

$$\mid \phi'(x') - \phi'(x'') \mid \leq L \mid x' - x'' \mid, x', x'' \in [a, b]. \tag{6}$$

New global optimization methods using a local information over the *whole* search region $[a, b]$ *during* the global search are proposed. The introduced approach is an alternative to traditional techniques which use local information only in a *neighborhood* of local minima *after* stopping some global procedures.

It is shown theoretically and numerically that the new approach gives a strong acceleration of the search and can be applied to a wide class of global optimization algorithms [3, 4, 5].

References

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