Numerical aspects in nonlinear diffusion equations for 2D/3D image and image sequence analysis

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Automatic image sequence restoration is a very important problem. Due to the fact that image sequence restoration is an emerging domain, the literature is not so abundant than the one related to the problem of restoring just a single image. Application areas include, for example, image surveillance, digital film restoration and medical image processing. We are particularly interested in two-dimensional (2D) and three-dimensional (3D) echocardiography, these are currently imaging modalities most frequently used in cardiology due to their simplicity, lack of ionizing radiation and a relative low cost. The correct visualization and interpretation of echo images is often affected by the high amount of noise intrinsically linked to the acquisition method. It is absolutely necessary to submit the data to pre-processing in order to improve their legibility from a clinical point of view. The application of traditional pre-processing algorithms (moving average, median and gaussian filtering) reduces the noise superimposed to the image but does not preserve a good definition of the details. Partial Differential Equation (PDE) models and methods have recently emerged in image processing as an alternative to traditional transform and statistically based methods [1],[3],[11],[12]. The PDEs are designed to possess certain desirable geometrical properties and allow a more systematic approach to both restoring images with sharp edges and image segmentation. The PDE formulations call for new computational techniques which are different from the traditional frequency domain and algebraic approaches. Among the computational difficulties that the new approach presents, there are the highly nonlinear and singular nature of the PDEs and the need to invert linear and nonlinear differential-integral operators efficiently.

The aim of this review is to present some properties of numerical schemes for nonlinear diffusion equations arising in image processing and computer vision. For this reason we have chosen two representative geometry driven diffusion models [5], namely the regularized Perona-Malik anisotropic diffusion equation [9], [3], the nonlinear degenerate diffusion equation of the mean curvature flow type [2] and three new models we have recently presented [10],[7], [6].

Our proposed nonlinear PDEs, representing the multiscale analysis, filter the sequence while keeping of the space-time coherent structures [10]. They combine the ideas of regularized Perona-Malik anisotropic diffusion [9] and Galilean invariant movie multiscale analysis of Alvarez, Guichard, Lions and Morel [1],[4]. The considered algorithms are able to distinguish the noise from the contours of the different structures by using in a distinct way spatial and temporal coherence.

The models present different properties and performances, hence they require different numerical techniques in order to derive stability properties, error estimates or convergence. However, all the models represent a similar view on image selective smoothing with keeping edge positions during multiscale analysis. The presented methods are based on semi-implicit approximation in scale and finite volume or finite elements in space and lead to the solution of linear systems of equations for each frame of the sequence at each discrete scale step. This can be done efficiently by using preconditioned iterative methods. It is clear that processing either high-resolution 2D/3D image sequences or very long sequences via PDE methods usually requires a significant number of iterations and memory resources. For example, for a 2D sequence with frame of 256x256 pixels, we have to compute quantities and solve linear systems involving 65536 unknowns and store many arrays of these dimensions. Furthermore, the development of 3D models would lead to problems of dimensions 256x256x256 that is 16777216 unknowns for each frame [10]. This precluds real-time processing without the use of parallel architectures. A preliminary parallel experimentation has been performed on Cray MPP T3E/1200 at CINECA Supercomputing Center using the Cray Message Passing Interface [8]. Computational results with artificial and real (echocardiographic) images are presented and discussed.

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