Newton method for Symmetric Inverse Eigenvalue Problems

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The inverse eigenvalue problem concerns the reconstruction of a matrix from prescribed spectral data. Inverse eigenvalue problems arise in different applications such as geophysics, particle phisics, circuit and control theory [1]. We restrict our attention to the *Symmetric Inverse Eigenvalue Problem* (SIEP), when a real symmetric matrix has to be constructed from a given set of eigenvalues. The problem can be formulated as follows.

Let $A_0, A_1, A_2, \ldots, A_n \in R^{n\times n}$ be real and symmetric matrices. Let be $c = (c_1, \ldots, c_n)^T \in R^n$. Let us define

$$A(c) = A_0 + \sum_{i=1}^{n} c_i A_i. \tag{1}$$

Let $\lambda_1^* < \lambda_2^* < \ldots < \lambda_n^*$ be real distinct eigenvalues, find c^* such that $A(c^*)$ has these eigenvalues.

Recently, the computational question of numerically constructing such a matrix has been accomplished in the case of structured matrices by using isospectral flows [1, 2].

A different approach to this problem has been investigated in [3] by solving a nonlinear system of equations $F(c) = 0$, with $F : R^n \to R^n$, suitably defined. Newton method is applied to the above system, solving SIEP in approximate way with locally quadratic convergence. Moreover, quasi-Newton method preserving quadratic convergence is used to reduce computational costs.

In this paper, we consider classical Newton method for two new choices of the function F involving traces of matrices. In both cases, our procedures are less expensive than quasi-Newton method in [3] and they are similarly stable.

A specific SIEP application is obtained by choosing A_0 and the basis A_i of matrices to fix the structure of $A(c)$. In particular, the additive and tridiagonal case are studied and numerical tests are given to show the performance of the presented approach.

References

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