A Numeric/Symbolic Tool in Total Hip Arthroplasty

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30 September 1999

In these last ten years, a continuous refinement of surgical and anaestesiological techniques has taken place, together with the introduction of new materials and new methods for treating the material itself and the finished products. This has allowed a swift rise in the number of total hip arthroplasty operations: in the sole of our country, more than twenty thousands operation per year are performed.

Monitoring the result of a surgical operation is of maximal importance, both to evaluate the condition of the prothesis itself, its successful stability, or to promptly diagnose the necessity for a correction. To meet this need, at the Rizzoli Orthopedical Institutes a software system is under development; in particular, the accurate detection of the *acetabular cup* through its radiographic evaluation, is under study and constitutes the motivation for this work.

We exploit the symbolic resources and the graphical capabilities of a system such as Mathematica, to simulate a problem model and validate numerical results obtained on real data. The model naturally leads to an ill-conditioned, overdetermined system, in which both the matrix and the know-term entries are affected by measurement errors. A C implementation of a least squares fitting method is then provided; results are reported, related to a set of real data problems, solved on a XP1000 CompaQ workstation.

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