ABS algorithms and ABSPACK

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The activity performed by the Bergamo unit has been mainly concerned with analysis, development and numerical experiments of ABS methods for linear systems and producing software based upon such methods. It is documented in several reports and papers.

Among the main new theoretical results we recall:

- generalization of the implicit LU algorithm to the case of arbitrary initial matrix, which is shown to play the role of a general right preconditioner. Since the ABS scaling parameter matrix plays the role of a general left preconditioner it follows that the ABS class is complete with respect to all possible preconditioners (i.e. choosing a preconditioner is equivalent to choosing some ABS parameters, a result which is rather unexpected);
- determination of the ABS parameters that allow to solve a general diophantine linear system for both a particular solution and the set of all solutions. This work generalizes a method Egervary proposed in 1955 for homogeneous systems and corrects an error in Egervary's paper. The ABS approach leads to a simple characterization of the necessary and sufficient conditions for the integer solvability, which is a natural generalization of the classical condition for the single equation in n variables;
- generalization of the ABS algorithms to linear matrix systems with application to the general solution of a Quasi-Newton equation in presence of sparsity and symmetry;
- further results on the alternative formulations of the implicit LU and LX methods in the framework of their use on the ABS reformulation of the simplex method. The ABS reformulation requires 8 times less memory and up to one order less operations (for problems with many constraints) than the usual formulation in terms of LU factors update (the Forrest-Goldfarb update);
- application of ABS methods to the computation of the inertia of a KKT matrix after changes induced by the active set strategy in the context of some new methods for constrained nonlinear optimization.

ABS methods have been implemented and experimented for general linear determined and underdetermined systems and general linear overdetermined systems solved in the least squares sense and for some classes of sparse structured problems. Particular attention has been paid to testing the accuracy on ill conditioned problems. The best algorithms have led to documented software. This work is expected to continue for some years and result in an ABSPACK package for linear and nonlinear algebraic systems and optimization problems.

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