Numerical Solution of some Problems in Tomography

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The problems arising in the Computed Tomography (CT) area are well known for their high dimensions and ill–posedness. Different CT problems such as X–ray Computed Tomography (TAC), Single Photon Emission Tomograpy (SPECT) and Positrons Emission Tomograpy (PET) are modeled by first kind Fredholm integral equations:

$$g(\theta, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K(x, y, \theta, t) f(x, y) dx dy \tag{1}$$

In practical problems the unknown function $f(x, y)$ represents different physical phenomena (*attenuation* in TAC, *emission* in SPECT and PET). The kernel $K(x, y, \theta, t)$ weights the contribution of each point of the object and models the data acquisition process. In this work we analize the kernel in the case of TAC and SPECT. In particular, we consider the problem of reconstructing human brain sections from experimental data obtained from a Gamma camera equipped with parallel–hole collimators [1],[3].

The discretization of equation (1) leads to a large size linear system. The high noise rates of the r.h.s. and the ill conditioning of the matrix require to compute a regularized least squares solution.

Both direct and iterative methods can be utilized [2] but, as shown in [4], iterative methods based on the conjugate gradient iterations are more efficient when applied to large size problems.

The introduction of regularization requires the determination of a suitable stopping criterion; this problem is addressed in [5] where different stopping criteria are compared.

In this work we report the results obtained in case of TAC and SPECT with simulated data (phantoms) and real acquisitions. We report also some details and results obtained with a parallel application running on Cray T3E.

References

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