Splitting and projection–type methods for large convex quadratic programs

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A well-known approach to the solution of large and sparse linearly constrained quadratic programming (QP) problems is given by the classical splitting and projection methods. These methods have a similar iterative scheme consisting in to solve a sequence of QP subproblems with the constraints of the original problem and an easily solvable Hessian matrix. When the structure of the constraints does not suggest a particular solver for the QP subproblems, we formulate each inner subproblem as a mixed linear complementarity problem (LCP) that can be solved by the classical projected SOR method of Cryer. When the number of constraints (and, consequently, the size of the inner LCPs) is large, appropriate parallel splitting methods for LCPs can be used for the solution on multiprocessor systems [3].

When the splitting of the Hessian matrix is suggested by the application in which the QP problem arises (as, for example, in the constrained bivariate interpolation problem [2]), the splitting method appears very efficient. In other cases, a convenient choice of the projection parameter determines a higher performance of the projection methods. If we multiply the Hessian matrix of each subproblem by a variable projection parameter, we generate a new method, called Variable Projection Method (VPM). For convex QP problems, the convergence of this scheme is obtained by introducing a simple contraction formula on the step length, without the typical restrictive conditions of the splitting and projection methods [5],[6]. An extensive numerical experimentation shows that the VMP combined with a practical nonexpensive updating rule for the projection parameter has an efficient behaviour in comparison with the classical splitting and projection methods, the recent modified projection-type methods and the NAG routine E04NKF [4]. A further application to the QP problem arising in training learning machines (named support vector machines [1]), emphasizes that the VPM is well suited to exploit particular structures in the matrices of the problem [7]. A variant of this method is presented in [6]. Here, the value suggested by the updating rule for the projection parameter is adaptively corrected in order to guarantee a sufficient decrease of the objective function and then the convergence of the scheme.

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